robKalman — a package on Robust Kalman Filtering

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16.06.2006

Classical setup: Linear state space models (SSMs)

- State equation:

$$\boxed{X_t = F_t X_{t-1} + v_t}$$

- Observation equation:

$$\boxed{Y_t = Z_t X_t + \varepsilon_t}$$

- Ideal model assumption:

$$X_0 \sim \mathcal{N}_p(a_0, \Sigma_0), \quad v_t \sim \mathcal{N}_p(0, Q_t), \quad \varepsilon_t \sim \mathcal{N}_q(0, V_t),$$

  all independent
- (preliminary ?) simplification: Hyper parameters F_t, Z_t, V_t, Q_t constant in t

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Problem and classical solution

- Problem: Reconststruction of X_t by means of $Y_s, s \leq t$
- Criterium: MSE
- $\rightsquigarrow$ general solution: $\mathrm{E}\, X_t | (Y_s)_{s \leq t}$
- Computational difficulties:
  $\Longrightarrow$ restriction to **linear** procedures
  / or: Gaussian assumptions
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Kalman filter

0. Initialization ($t = 0$):

$$X_{0|0} = a_0, \quad \Sigma_{0|0} = \Sigma_0$$

1. Prediction ($t \geq 1$):

$$X_{t|t-1} = FX_{t-1|t-1}, \quad \mathrm{Cov}(X_{t|t-1}) = \Sigma_{t|t-1} = F\Sigma_{t-1|t-1}F' + Q$$

2. Correction ($t \geq 1$):

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + K_t(Y_t - ZX_{t|t-1}) \\ K_t &= \Sigma_{t|t-1}Z'(Z\Sigma_{t|t-1}Z' + V)^-, \qquad \text{(Kalman gain)} \\ \mathrm{Cov}(X_{t|t}) &= \Sigma_{t|t} = \Sigma_{t|t-1} - K_t Z \Sigma_{t|t-1} \end{aligned}$$

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Types of outliers and robustification

- IOs (system intrinsic): state equation is distorted
  — not considered here
- AO/SOs (exogeneous): observations are distorted:
  - either error ε_t is affected (AO)
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- a robustifications as to AO/SOs is to
  - retain recursivity (three-step approach)
  - modify correction step $\rightsquigarrow$ bound influence of Y_t
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Considered approaches

Approximate conditional mean (ACM): [Martin(79)]

- dim $Y_t = 1$
- particular model: $Y_t \sim \mathrm{AR}(p)$
  - $\rightsquigarrow X_t = (Y_t, \ldots, Y_{t-p+1})$,
  - hyper parameters $Z = (1, 0, \ldots, 0)$, $V^{\mathrm{id}} = 0$, F, Q unknown
- estimation of F, Q by means of GM-Estimators
- modified Corr.step: for suitable location influence curve ψ

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + \Sigma_{t|t-1} Z' \psi(Y_t - Z X_{t|t-1}) \\ \Sigma_{t|t} &= \Sigma_{t|t-1} - \Sigma_{t|t-1} Z' \psi'(Y_t - Z X_{t|t-1}) Z \Sigma_{t|t-1} \end{aligned}$$

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Concept and strategy

Goal: package `robKalman`

Contents

- Kalman filter: filter, Kalman gain, covariances
- ACM-filter: filter, GM-estimator
- rLS-filter: filter, calibration of clipping height
- further recursive filters?
  $\rightsquigarrow$ general interface `recursiveFilter`
  with arguments:
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  - completely in `S`
  - perhaps some code in `C` (much) later
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    - univariate setting: `KalmanLike` (package `stats`);
      time series classes: `ts`, `its`, `irts`, `zoo`, `zoo.reg`, `tframe`
    - multivariate setting: `dse` bundle by Paul Gilbert; perhaps `zoo`?
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  - internal functions: no S4-objects
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Concept and strategy III

- Use of `S4`
  - Hierarchic Classes:
    - state space models (SSMs) (Hyper-Parameter, distributional assumptions, outlier types)
    - filter results (specific subclass of (multivariate) time series)
    - control structures for filters (tuning parameters)
  - Methods:
    - filters (for different types of SSMs)
    - accessor/replacement functions
    - `simulate` for SSMs
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- preliminary, "S4-free" interfaces
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    - with function `arGM` for AR-parameters by GM-estimates
    - various ψ-functions are available:
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    ⇝ close contact to
    - `RCore`,
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  - casting/conversion functions for various time series classes
- User interface `robfilter` (?)
  - goal: four arguments: data, SSM, control-structure, filter type
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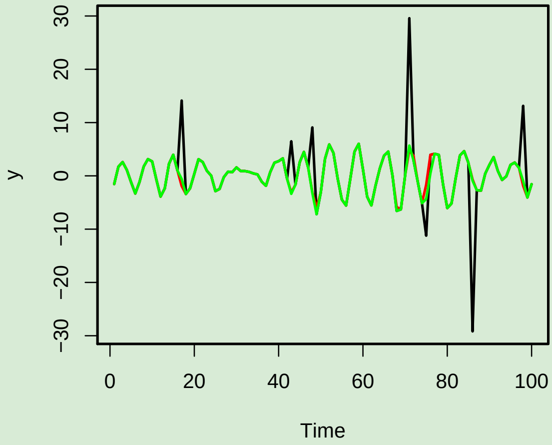
Demonstration: ACMfilt

```
## generation of data from AO model:
set.seed(361)
Eps <- as.ts(rnorm(100))
ar2 <- arima.sim(list(ar = c(1, -0.9)),
       100, innov = Eps)
Binom <- rbinom(100, 1, 0.1)
Noise <- rnorm(100,sd = 10)
y <- ar2 + as.ts(Binom*Noise)

## determination of GM-estimates
y.arGM <- arGM(y, 3)
## ACM-filter
y.ACMfilt <- ACMfilt(y, y.arGM)

plot(y)
lines(y.ACMfilt$filt, col=2)
lines(ar2,col="green")
```



green: ideal time series,
black: AO contam. time series,
red: result ACM

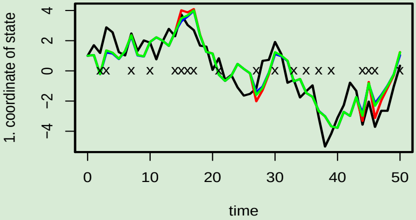
Demonstration: rLSFilter

```
## specification of SSM: (p=2, q=1)
a0   ← c(1, 0); S0  ← matrix(0, 2, 2)
F    ← matrix(c(.7, 0.5, 0.2, 0), 2, 2)
Q    ← matrix(c(2, 0.5, 0.5, 1), 2, 2)
Z    ← matrix(c(1, −0.5), 1, 2)
Vi   ← 1;
## time horizon:
TT←50
## AO−contamination
mc   ← −20; Vc  ← 0.1; ract ← 0.1
## for calibration
r1←0.1; eff1←0.9

#Simulation::
X    ← simulateState(a, S0, F, Q, TT)
Yid  ← simulateObs(X, Z, Vi, mc, Vc, r=0)
Yre  ← simulateObs(X, Z, Vi, mc, Vc, ract)
```

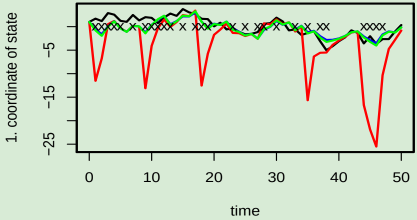
Demonstration: rLSfilter II

```
### calibration b
#limiting S_{t|t-1}
SS <- limitS(S, F, Q, Z, Vi)
# by efficiency in the ideal model
(B1 <- rLScalibrateB(eff=eff1, S=SS, Z=Z, V=Vi))
# by contamination radius
(B2 <- rLScalibrateB(r=r1, S=SS, Z=Z, V=Vi))

### evaluation of rLS
rerg1.id <- rLSFilter(Yid, a, Ss, F, Q, Z, Vi, B1$b)
rerg1.re <- rLSFilter(Yre, a, Ss, F, Q, Z, Vi, B1$b)
rerg2.id <- rLSFilter(Yid, a, Ss, F, Q, Z, Vi, B2$b)
rerg2.re <- rLSFilter(Yre, a, Ss, F, Q, Z, Vi, B2$b)
```

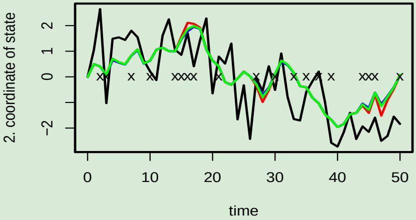
ideal situation

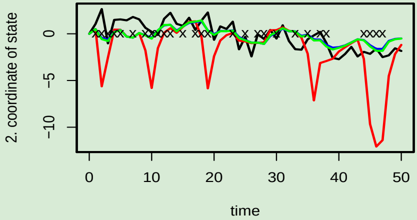


AO-contaminated situation



black:	real state,	green:	rLS filter (B1),
red:	class. Kalman filter	blue:	rLS filter (B2)





Bibliography

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